Flexibility and coordination in a supply chain with bidirectional option contracts and service requirement

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Abstract: Option contracts have been increasingly employed by supply chain firms as a popular strategy to hedge against the risk of unanticipated demand. This paper examines the impact of bidirectional option contracts on a two-echelon supply chain consisting of a supplier and a retailer taking into consideration of a service requirement. We characterize the optimal solutions for the retailer and the supplier with and without bidirectional option contracts in the presence of a service requirement. By benchmarking to the model without bidirectional option, we explore the effect of bidirectional option contracts on the supply chain. Our study shows that the service level with bidirectional option contracts is equivalent to (higher than) that without them when the service requirement is (not) binding. In addition, bidirectional option contracts are indeed beneficial to both the retailer and the supplier. Furthermore, by investigating the effect of the service level on the supply chain, we find that the maximum expected profit of the retailer is non-increasing in the service requirement while that of the supplier is non-decreasing in the service requirement, either with or without bidirectional option contracts. Finally, a distribution-free coordination condition is proposed to achieve the Pareto improvement in the presence of bidirectional option contracts and a service requirement.

Keywords: Flexibility; Bidirectional option contracts; Supply chain risk management; Coordination; Service requirement

1. Introduction

With the hastened technology advancement, an increasing number of products are characterized with a relatively shorter life cycle. This phenomenon is particularly prominent in automobile industry, electronics industry and high-tech industry. Under such a situation, consumer taste changes at a fast pace and so an accurate prediction of the market demand becomes more difficult. To accommodate to the changing market environment, companies need to equip with the capability of responding flexibly to high demand uncertainty. A typical approach is to use one type of contracts that give the retailer the flexibility in ordering products without imposing a burden on the supplier. Among all the contracts, option contracts can effectively resolve the conflicts of interest between the supplier and the retailer described above. With option contracts, the initial order of the retailer is allowed to be adjusted after the market demand is revealed and the supplier can carry out the production schedule more flexibly. Over the past years, option contracts are gradually adopted by many well-known companies as a popular strategy in procurement. For example, option contracts are adopted by Hewlett-Packard to purchase various components such as memory chips and scanner, which account for 35% procurement value (Fu *et al.*, 2010). Most of the products such as mobile phone and relevant accessories, which value more than 100 billion RMB, are purchased by China Telecom through using option contracts (Chen and Shen, 2012). Option contracts have been demonstrated to be a viable alternative to hedge against the risk of unanticipated demand.

Usually option contracts can be classified into two different types: unilateral and bidirectional. Unilateral option contracts are either call or put option contracts. For call option contracts, the retailer can reorder a certain quantity of items from the supplier if necessary and the retailer’s initial order can be adjusted in the upward direction. For put option contracts, the retailer can return a certain quantity of items to the supplier if necessary and the retailer’s initial order can be adjusted in the downward direction. However, both call and put options are contained in the bidirectional option contracts, for which, the initial order of the retailer can be adjusted in two directions. Obviously, they are an extension of unilateral option contracts (Moinzadeh and Nahmias, 2013) and its adoption can provide more flexibility to the retailer in responding to the demand volatility. In recent years, bidirectional option contracts have been widely employed in many industry sectors in including agricultural products, automobiles and electronics. Recently, through our fieldwork in Hainan province of China, we found that some companies specializing in the transaction of flowers launch bidirectional option contracts to distribute the flowers to the global customers. Then the customers obtain the flowers by the initial order or exercising bidirectional options based on the pre-negotiation. However, companies encountered many issues related to bidirectional option contracts, which have not yet been resolved. Motivated by this, this paper attempts to address some important issues of bidirectional option contracts.

Meanwhile, with the increasing market competition, a required service level becomes one of the key factors that exert an impact on the purchase choice of consumers. Through promising a service level, companies can maintain the market share in the existing market and gain a competitive edge in the new market. Many companies are willing to promise a high service level in order to promote the sales of products. For example, Pharmed Group (PMG) guarantees that the fill rate of their customer orders is no less than 98% (PMG 2006). Costless Express promises that its customers can receive their order on the next-business-day (Costless Express 2006). However, the higher service target the companies commit to, the higher risk the companies have to undertake. Therefore, how to set an appropriate service target is critical for a company to balance the customer satisfaction and the expected profit. Motivated by this, the service level related issues are also incorporated in this study.

In this paper, the procurement and production problems are discussed in a supply chain setting that one supplier manufactures one type of products with limited product life and distributes them to the market via one retailer. To enhance customer satisfaction, the retailer commits to a service target that is provided to the customers. Moreover, to hedge against the risk associated to uncertain demand, bidirectional option contracts are employed by the retailer when purchasing products from the supplier. This study addresses the problem referring to the optimal operational decisions and the maximum expected profits of the retailer and the supplier in the presence of bidirectional option contracts and a required service level. Several key questions are address in this paper:

- How do bidirectional option contracts affect the supply chain?
- What effect does the service level have on the supply chain?
- What is the condition for coordinating the supply chain with bidirectional option contracts and required service level?

The remainder of our paper is organized as follows. A review of option contracts and the service requirement related literature is provided in section 2. The model assumptions and formulation are provided in section 3. In the next section, we derive the optimal decision policies for both the retailer and the supplier respectively with bidirectional option contracts and a required service level. In section 5, we discuss the effect of bidirectional option contracts and the service level requirement on the supply chain. In section 6, we examine the condition for the supply chain coordination in the presence of bidirectional option contracts and a required service level. In section 7, some directions for further research are provided.

2. Literature review

To highlight the work of this paper, the related literature is reviewed in two important research streams: (1) option contracts and (2) models with a service requirement.

The research referring to option contracts has drawn considerable attentions in the past 15 years. Barnes-Schuster *et al.* (2002) introduce real options into the supply chain management research for the first time. They prove that the two-period channel coordination with correlated demand is achievable in the presence of call option contracts when the exercise price is restricted to be linear. Burnetas and Ritchken (2005) analyze the change that occurs on both the wholesale price and the retail price after using call option contracts when consumers have a downward sloping demand. Wang and Tsao (2006) study the optimal behavior of the buyer with bidirectional option contracts. They prove that the adoption of bidirectional option contracts can enhance the buyer’s profitability under the condition of uniformly distributed demand. Wang and Liu (2007) analyze the coordination issue of a supply chain with call option contracts when the dominant position of the market is taken by the retailer. Gomez_Padilla and Mishina (2009) show that bidirectional option contracts benefit both the members and the chain where multiple suppliers and one retailer are the members. They also obtain the same results for a supply chain that only includes one supplier and one retailer. Fu *et al.* (2010) study the single-period portfolio procurement problem with call option contracts and spot market, and then extend to the case of multi-period procurement. Xia *et al.* (2011) explore how to employ option and firm order contracts to share the risks of supply and demand. They demonstrate that the reliable supplier is always preferred by the buyer and placing pre-orders can improve the supplier’s operations. Liu *et al.* (2013) introduce both unilateral and bidirectional option contracts into the container planning problem. They study the application strategies of different option contracts in different practical scenarios. Chen *et al.* (2014) investigate the coordination issue of a supply chain considering a loss-averse retailer and call option contracts. All of the related literature mainly focuses on call option contracts and the literature on bidirectional option contracts is relatively rare. Moreover, all of the related literature does not consider the service requirement and its impacts.

The research referring to models with a service requirement has also inspired numerous studies over the past years. Bernstein and Federgruen (2007) develop a mechanism that combines wholesale price contracts and back-logging penalties to coordinate a supply chain, in which, customer demand is dependent on price and service level. Sethi *et al.* (2007) analyze the problem of the buyer's procurement with a service target and demand forecast updating. They show that the coordination of the supply chain is available in the presence of buyback contracts. Katok *et al.* (2008) demonstrate that the supply chain can be coordinated by service-level commitment contracts. They also explore the impact of the review period length and the impact of the bonus quantity on the optimal inventory level. Li *et al.* (2011) derive the optimal solutions of the two inventory optimization models with a required service level and then propose a price discount mechanism to acheve supply chain coordination. Sieke *et al.* (2012) propose two types of supply contracts with a service requirement to coordinate the partners' activities within a supply chain. Xiao and Xu (2013) obtain the optimal solutions referring to the price and the service level and then develop a revenue-sharing mechanism for the supply chain coordination under vendor-managed inventory (VMI). Jha and Shanker (2013) present a production-inventory model of a supply chain that is made of one vendor and multiple buyers, taking into consideration of lead time reduction and service level constraints. Heydari (2014) investigates the relationship between the supplier's lead time and the chain's service level. They develop a coordination mechanism with per order extra payment to coordinate the supplier's replenishment decision with stochastic lead time and a service requirement. All of the related literature does not consider option contracts and its impacts.

Our work is an extension of the work of Zhao *et al.* (2013) and Chen and Shen (2012). In analyzing the optimal ordering strategy of the retailer with bidirectional option contracts, Zhao et al. (2013) provide the condition on which the supply chain can be coordinated. However, their study only focuses on the retailer's ordering behavior. Since the supplier is assumed to manufacture the items up to the retailer's order quantity, the supplier's production behavior is not considered in their paper. Moreover, their study only examines the impact of bidirectional option contracts on the retailer's initial order but never explore its impact on the retailer's maximum expected profit, let alone its impact on the supplier's maximum expected profit. Furthermore, their study does not incorporate the service requirement into their model. Chen and Shen (2012) demonstrate that the adoption of call option contracts can improve the expected profits for the two members when there is a service requirement. They also explore the effect of the service level on the maximum expected profits for both the retailer and the supplier in the presence of call option contracts. However, they research only considers call option contracts. As we know, bidirectional options contracts are regarded as an extension of unilateral options contracts. A series of issues involving bidirectional option contracts have not been addressed. Different from these above research, this paper incorporate bidirectional option contracts and service requirement into the model. We analyze the optimal decision policies for both the retailer and the supplier with bidirectional option contracts and a required service level. We also investigate the effect of the above two factors on the supply chain. On the basis, we derive the condition on which the supply chain can achieve coordination in the presence of bidirectional option contracts and a required service level.

3. Model assumptions and formulation

This paper considers a one-period two-echelon supply chain, in which the supplier manufactures a type of short life products and sells them via the retailer. The short life products, subjecting to uncertain further market demand, are characterized by a long lead-time and a short selling period. For this reason, the retailer is not able to replenish his inventory during the selling season. Moreover, in order to promote the sales of products, the retailer commits to a service target to the consumers. For this reason, the retailer should order enough quantity to enhance the probability of satisfying the market demand.

The variables and model parameters are defined as the notations presented in Table 1.

Tab. 1 Notations

Notation	Descriptions
D	The stochastic market demand.
$f(x)$	Probability density function for D.
$F(x)$	Distribution function for D, which is differentiable, invertible and strictly increasing.
p	Unit retail price.
w_1	Unit wholesale price of the product.
b	Unit purchase price of the bidirectional option.
w_2	Unit exercise price of the bidirectional option.
c	Unit production cost.
s	Units salvage value.
g	Supplier's unit penalty cost for each exercised bidirectional option which cannot be immediately satisfied
α	Retailer's service level commitment. Note $0 < \alpha \leq 1$.
q_1	Retailer's firm order quantity with bidirectional options contracts.
q_0	Retailer's firm order quantity without bidirectional options contracts.
q_2	Retailer's bidirectional options order quantity.
q	Retailer's total order quantity with bidirectional options contracts.
Q	Supplier's production quantity with bidirectional options contracts.
Q_0	Supplier's production quantity without bidirectional options contracts.
$\pi_r(q_1, q_2)$	Retailer's expected profit with bidirectional options contracts and without a service requirement.
$\pi_r(q_0)$	Retailer's expected profit without bidirectional options contracts and a service requirement.
$\pi_s(Q)$	Supplier's expected profit with bidirectional options contracts and without a service requirement.
$\pi_s(Q_0)$	Supplier's expected profit without bidirectional options contracts.

q_1^{γ}	Retailer's optimal firm order quantity with bidirectional options contracts and without a service requirement.
q_0^{β}	Retailer's optimal firm order quantity without bidirectional options contracts and a service requirement.
q_2^{γ}	Retailer's optimal bidirectional options order quantity without a service requirement.
q^{γ}	Retailer's optimal total order quantity with bidirectional options contracts and without a service requirement. Note, $q^{\gamma} = q_1^{\gamma} + q_2^{\gamma}$.
Q^{λ}	Supplier's optimal production quantity with bidirectional options contracts and without a service requirement.
q_2^{*}	Retailer's optimal bidirectional options order quantity with a service requirement.
q^{*}	Retailer's optimal total order quantity with bidirectional options contracts and a service requirement.
q_0^{*}	Retailer's optimal firm order quantity without bidirectional options contracts and with a service requirement.
Q^{*}	Supplier's optimal production quantity with bidirectional options contracts and a service requirement. Note, $q_1^{\gamma} < Q^{*} < q^{*}$.
Q_0^{*}	Supplier's optimal production quantity without bidirectional options contracts and with a service requirement. Note, $Q_0^{*} = q_0^{*}$.

The decision process of the model with bidirectional options contracts is illustrated in Figure 1. Before the start of the selling season, the retailer places an initial order q_1 at unit wholesale price w_1 and purchases a certain amount of bidirectional options q_2 at unit purchase price b. Each bidirectional option provides the retailer with the right, not the obligation, to reorder or return one extra item after the market demand has been observed. After that, the supplier produces the products up to Q based on the retailer's order quantity and the stochastic demand. At the start of the selling season, the firm order is delivered immediately to the retailer. During the selling season, the retailer exercises the bidirectional options as either the call options or the put options based on the realized market demand at unit exercise price w_2. Any leftovers owned by either the retailer or the supplier can be salvaged at the end of the selling period.

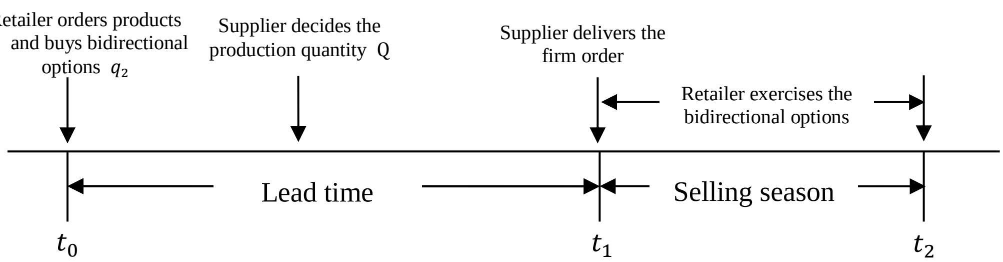

Fig. 1 The decision process with bidirectional options contracts

The supply chain members are assumed to be rational and neutral. The information available is symmetric between the two parties. All the bidirectional options are settled by physical delivery of product rather than cash payment. This assumption ensures that the retailer has to consider the realized market demand and never exercises all the bidirectional options just to gain benefit from the penalty costs. Thus, the unit penalty cost g denotes the cost involved in accelerating production or obtaining products from an alternative source.

To avoid the trivialities, the contract parameters are assumed to satisfy the following conditions throughout this paper.

(1) $w_1 - b < w_2 < w_1 + b$. The first part of this condition ensures that the costs of obtaining one products are higher with bidirectional option contracts than without them because bidirectional option contracts provide the retailer flexibility in responding to the market variation. Otherwise, the retailer has no motivation to use wholesale price contracts to obtain the goods from the supplier. The second part of this condition avoids that the retailer deliberately orders more bidirectional options. Otherwise, it is always profitable for the retailer to purchase bidirectional options.

(2) $p - w_2 > w_2 - s$. This condition ensures that the retailer must satisfy the market when the demand occurs, not return the products to the supplier.

(3) $p - c > c - s$. This condition ensures that the opportunity revenue is greater than the opportunity loss. Otherwise, the supplier has no incentive to manufacture the products.

(4) $g > c > s$. The fourth condition ensures that the unit production cost is lower than the unit penalty cost, but higher than the unit salvage value.

(5) In our study, similar to Zhao *et al.* (2013), the allowance paid for exercising one unit of bidirectional option as one unit of call option is supposed to be the same as the refund obtained from exercising one unit of bidirectional option as one unit of put option. Notation $x^+ = \max(0, x)$.

4. Optimal decisions with bidirectional option contracts

In this section, a model with bidirectional option contracts is formulated and the optimal ordering and production policies are analyzed for both the retailer and the supplier.

4.1 Optimal ordering policy with bidirectional option contracts

First, the optimal ordering policy of the retailer is analyzed with bidirectional option contracts and a service requirement. In this context, two decision variables are considered including: the firm order quantity q_1 and the bidirectional option order quantity q_2.

With bidirectional option contracts, the retailer's expected profit without a service requirement is

$$\pi_r(q_1, q_2) = pE[\min(q_1 + q_2, D)] + sE[(q_1 - q_2 - D)^+] + w_2E[\min(q_2, (q_1 - D)^+)]$$
$$- w_2E[\min(q_2, (D - q_1)^+)] - bq_2 - w_1q_1$$

The first three terms are the expected revenue, the salvage value of unsold products and the incomes realized from exercising the bidirectional options as the put options, respectively. The last three terms capture the costs of exercising the bidirectional options as the call options, purchasing bidirectional options and ordering the products, respectively. Then,

$$\pi_r(q_1, q_2) = (p - w_1)q_1 + (p - w_2 - b)q_2 - (p - w_2)\int_0^{q_1+q_2} F(x)dx$$

$$-(w_2 - s)\int_0^{q_1-q_2} F(x)dx \tag{1}$$

Thus, the decision problem faced by the retailer with bidirectional option contracts and a service requirement is

$$\max_{q_1 \geq 0, q_2 \geq 0} \pi_r(q_1, q_2)$$

$$s.t.\ P_r\{q \geq D\} \geq \alpha \tag{2}$$

From (2), we derive that $q \geq F^{-1}(\alpha)$ and $q^{\alpha} \equiv F^{-1}(\alpha)$. It's clear that q^{α} is an increasing function of α.

Lemma 1 ***With bidirectional option contracts, the retailer's expected profit without a service requirement*** $\boldsymbol{\pi_r(q_1, q_2)}$ ***is jointly concave in*** $\boldsymbol{q_1}$ ***and*** $\boldsymbol{q_2}$***.***

From this lemma, we conclude that without a service requirement the retailer has a unique optimal ordering policy under portfolio contracts with bidirectional options. Then, with bidirectional option contracts the retailer's optimal firm order quantity without a service requirement is

$$q_1^{\gamma} = \frac{1}{2}\left[F^{-1}\left(\frac{2p-w_1-w_2-b}{2p-2w_2}\right) + F^{-1}\left(\frac{w_2+b-w_1}{2w_2-2s}\right)\right] \tag{3}$$

The retailer's optimal bidirectional options order quantity without a service requirement is

$$q_2^{\gamma} = \frac{1}{2}\left[F^{-1}\left(\frac{2p-w_1-w_2-b}{2p-2w_2}\right) - F^{-1}\left(\frac{w_2+b-w_1}{2w_2-2s}\right)\right] \tag{4}$$

The retailer's optimal total order quantity without a service requirement is

$$q^{\gamma} = F^{-1}\left(\frac{2p-w_1-w_2-b}{2p-2w_2}\right) \tag{5}$$

The above results are constant with Theorem 1 of Zhao *et al.* (2013) except that the lost sales penalty is not considered here. Set $\gamma = (2p - w_1 - w_2 - b)/(2p - 2w_2)$, which represents the maximum service level with bidirectional option contracts when there is no service requirement. Then, we obtain that $q^{\gamma} \equiv F^{-1}(\gamma)$.

Since $q^{\gamma} > q_1^{\gamma}$, we obtain that $b < \frac{(p-w_1)(w_2-s)+(p-w_2)(w_1-s)}{p-s}$. This inequality implies that if the supplier charges a high purchase price of bidirectional options, the retailer will never purchase any number of bidirectional options.

Regarding the retailer's optimal ordering policy with bidirectional option contracts and a service requirement, we have the following proposition.

Proposition 1 ***With bidirectional option contracts, the retailer's optimal total order quantity with a service requirement satisfies***

$$\boldsymbol{q^* = \begin{cases} q^{\gamma} & if\ \alpha \leq \gamma \\ q^{\alpha} & if\ \alpha > \gamma \end{cases}} \tag{6}$$

and his optimal bidirectional options order quantity with a service requirement is

$$\boldsymbol{q_2^* = q^* - q_1^{\gamma}} \tag{7}$$

This proposition shows that there is an obvious difference between the retailer's optimal ordering policies either with or without a service requirement considering bidirectional option contracts. If $\alpha \leq \gamma$ then $q^* = q^{\gamma}$ and $q_2^* = q_2^{\gamma}$. If $\alpha > \gamma$ then $q^* = q^{\alpha}$ and $q_2^* \neq q_2^{\gamma}$.

Now we explore the effect of w_1 on the retailer's maximum expected profit with bidirectional option contracts and a service requirement, which brings about the following corollary.

Corollary 1 ***With bidirectional option contracts, the retailer's maximum expected profit with a service requirement is a decreasing function of*** $\boldsymbol{w_1}$**.**

This corollary shows that considering bidirectional option contracts and a service requirement, the retailer's maximum expected profit is significantly influenced by the changes in the unit wholesale price. If the unit wholesale price increases, the retailer will generate a lower expected profit with bidirectional option contracts and a service requirement. Otherwise, the retailer will gain a higher expected profit.

4.2 Optimal production policy with bidirectional option contracts

Subsequently, we consider the optimal production policy of the supplier with bidirectional option contracts and a service requirement. Since the make-to-order production pattern is not adopted by the supplier, with bidirectional option contracts and a service requirement, the supplier's production quantity satisfies $q_1^{\gamma} < Q < q^*$.

With bidirectional option contracts, the supplier's expected profit without a service requirement is

$$\pi_s(Q) = w_1 q_1^{\gamma} + b\left(q^* - q_1^{\gamma}\right) + sE\left[\min\left(Q - (2q_1^{\gamma} - q^*), (Q - x)^+\right)\right] + w_2 E\left[\min\left(q^* - q_1^{\gamma}, \left(D - q_1^{\gamma}\right)^+\right)\right] - w_2 E\left[\min\left(q^* - q_1^{\gamma}, \left(q_1^{\gamma} - D\right)^+\right)\right] - cQ - gE[(\min(D, q^*) - Q)^+]$$

The first four terms capture the income s realized from the firm order, the bidirectional options order, the salvaged unsold product and exercising the bidirectional options as the put options, respectively. The last three terms are the costs of exercising the bidirectional options as the call options, the production costs and the penalty costs, respectively. Then,

$$\pi_s(Q) = (w_2 + b - g)q^* + (w_1 - w_2 - b)q_1^\gamma - (w_2 - g)\int_0^{q^*} F(x)\,dx$$
$$+(w_2 - s)\int_0^{2q_1^\gamma - q^*} F(x)\,dx + (g - c)Q - (g - s)\int_0^{Q} F(x)dx \qquad (8)$$

Thus, the decision problem faced by the supplier with bidirectional option contracts and a service requirement is

$$\max_Q \pi_s(Q)$$
$$s.t.\;\; q_1^\gamma \le Q \le q^* \qquad (9)$$

Lemma 2 ***With bidirectional option contracts, the supplier's expected profit without a service requirement*** $\boldsymbol{\pi_s(Q)}$ ***is concave in*** $\boldsymbol{Q}$***.***

From this lemma, we conclude that without constraints the supplier has a unique optimal production policy under portfolio contracts with bidirectional options. Then, with bidirectional option contracts the supplier's optimal production quantity Q^λ without a service requirement is

$$Q^\lambda = F^{-1}\left(\frac{g-c}{g-s}\right) \qquad (10)$$

Set $\lambda = (g - c)/(g - s)$. Then, we obtain that $Q^\lambda \equiv F^{-1}(\lambda)$.

As to the supplier's optimal production policy with bidirectional option contracts and a service requirement, the following proposition is derived.

Proposition 2 ***With bidirectional option contracts, the supplier's optimal production quantity with a service requirement satisfies***

$$\boldsymbol{Q^*} = \begin{cases} \boldsymbol{q_1^\gamma} & \boldsymbol{if\; Q^\lambda \le q_1^\gamma} \\ \boldsymbol{Q^\lambda} & \boldsymbol{if\; q_1^\gamma < Q^\lambda < q^*} \\ \boldsymbol{q^*} & \boldsymbol{if\; Q^\lambda \ge q^*} \end{cases} \qquad (11)$$

This proposition shows that with bidirectional option contracts and a service requirement, the supplier's optimal production quantity is an interval. If $Q^\lambda \le q_1^\gamma$ then $Q^* = q_1^\gamma$. If $q_1^\gamma < Q^\lambda \le q^*$ then $Q^* = Q^\lambda$. If $\alpha \le \gamma$ and $q^* \le Q^\lambda$ then $Q^* = q^\gamma$. If $\alpha > \gamma$ and $q^* \le Q^\lambda$ then $Q^* = q^\alpha$.

Now we explore the effect of w_1 on the supplier's maximum expected profit with bidirectional option contracts and a service requirement, which leading to the following corollary.

Corollary 2 ***When $\alpha > \gamma$, the supplier's maximum expected profit with bidirectional option contracts and a service requirement is an increasing function of w_1.***

This corollary shows that when the service requirement is binding ($\alpha > \gamma$), the supplier's maximum expected profit with bidirectional option contracts will increase as the unit wholesale price increases. Recalling corollary 1, we conclude that when the service requirement is binding, an increase in the unit wholesale price will induce a rise in the supplier's maximum expected profit and the decrease in that of the retailer with bidirectional option contracts.

5. Discussion

5.1 The case without bidirectional option contracts

In this section, we formulate a model without bidirectional option contracts and then take this model as a benchmark to explore the effect of bidirectional option contracts and the service level on the supply chain.

We assume that the retailer only places a firm order q_0 from the supplier. Without bidirectional option contracts and a service requirement, the retailer's expected profit is

$$\pi_r(q_0) = pE[\min(q_0, D)] + sE[(q_0 - D)^+] - w_1 q_0$$

The first two terms are the expected revenue and the salvage value of unsold products. The last term is the costs of ordering the products. Then,

$$\pi_r(q_0) = (p - w_1)q_0 - (p - s)\int_0^{q_0} F(x)dx \tag{12}$$

Thus, the decision problem faced by the retailer without bidirectional option contracts in the presence of service requirement is

$$\max_{q_0 \geq 0} \pi_r(q_0)$$

$$s.t.\ P_r\{q_0 \geq D\} \geq \alpha \tag{13}$$

From (13), we derive that $q_0 \geq F^{-1}(\alpha)$ and $q^\alpha \equiv F^{-1}(\alpha)$. It is clear that q^α is an increasing function of α.

From (12), we obtain that $\frac{d\pi_r(q_0)}{dq_0} = (p - w_1) - (p - s)F(q_0)$ and $\frac{d^2\pi_r(q_0)}{dq_0^2} = -(p - s)f(q_0) < 0$. Obviously, $\pi_r(q_0)$ is concave in q_0. Hence, we conclude that without a service requirement the retailer has a unique optimal ordering policy under wholesale price contracts. Then, without bidirectional option contracts and as service requirement, the retailer's optimal firm order quantity is

$$q_0^{\beta} = F^{-1}\left(\frac{p-w_1}{p-s}\right) \tag{14}$$

Set $\beta = (p - w_1)/(p - s)$, which represents the maximum service level without bidirectional option contracts when there is no service requirement. Then, we obtain that $q_0^{\beta} \equiv F^{-1}(\beta)$.

Without bidirectional option contracts, the retailer's optimal firm order quantity in the presence of a service requirement is

$$q_0^* = \begin{cases} q_0^{\beta} & if\ \alpha \leq \beta \\ q^{\alpha} & if\ \alpha > \beta \end{cases} \tag{15}$$

Recalling (13), without bidirectional option contracts the retailer's maximum expected profit in the presence of a service requirement is $\pi_r(q_0^*) = (p - w_1)q_0^* - (p - s)\int_0^{q_0^*} F(x)dx$. When $\alpha \leq \beta$, we obtain that $q_0^* = q_0^{\beta}$. In this case, we derive that $\pi_r(q_0^*)$ is a constant function of α. When $\alpha > \beta$, we obtain that $q_0^* = q^{\alpha}$. In this case, we derive that $\pi_r(q_0^*)$ is a decreasing function of α. In conclusion, without bidirectional option contracts the retailer's maximum expected profit in the presence a service requirement is a non-increasing function of α.

We assume that the supplier's production quantity is Q_0. When the selling season begins, the supplier must deliver the firm order to the retailer. Hence, the supplier's optimal production quantity satisfies $Q_0^* = q_0^*$. Without bidirectional option contracts, the supplier's expected profit in the presence of a service requirement is

$$\pi_s(Q_0^*) = (w_1 - c)Q_0^* = (w_1 - c)q_0^* \tag{16}$$

When $\alpha \leq \beta$, we obtain that $Q_0^* = q_0^{\beta}$. In this case, we derive that $\pi_s(Q_0^*)$ is a constant function of α. When $\alpha > \beta$, we obtain that $Q_0^* = q^{\alpha}$. In this case, we derive that $\pi_s(Q_0^*)$ is a increasing function of α. In conclusion, without bidirectional option contracts the supplier's maximum expected profit in the presence of a service requirement is a non-decreasing function of α.

5.2 The effect of bidirectional option contracts

First, we consider the effect of bidirectional option contracts on the retailer's optimal ordering policy, which yields the following proposition.

Proposition 3 ***If*** $\boldsymbol{\alpha \leq \gamma}$***, then*** $\boldsymbol{q^* > q_0^*}$***. If*** $\boldsymbol{\alpha > \gamma}$ ***then*** $\boldsymbol{q^* = q_0^*}$**.**

This proposition shows that the introduction of bidirectional option contracts has a remarkable impact on the retailer's optimal total order quantity. When the service requirement is not binding ($\alpha \leq \gamma$), the retailer's optimal total order quantity is greater with bidirectional option contracts than without them, meaning that the service level should be improved to a higher level after adopting bidirectional option contracts. When the service requirement is binding ($\alpha > \gamma$), the retailer's optimal total order quantity with bidirectional option contracts is equivalent to that without them, meaning that the service level should be still maintained at the original level after adopting bidirectional option contracts. It follows that the usage of bidirectional option contracts can help the retailer to improve the service level when there is no service requirement and maintain the service level when there is a service requirement.

We depict this proposition in figure 2, in which, β represents the maximum service level corresponding to the retailer's optimal firm order quantity without bidirectional option contracts and a service requirement. γ represents the maximum service level corresponding to the retailer's optimal total order quantity with bidirectional option contracts and without a service requirement. Moreover, we see that γ is always greater than β when there is no service requirement, which implies that the maximum service level is higher with bidirectional option contracts than without them.

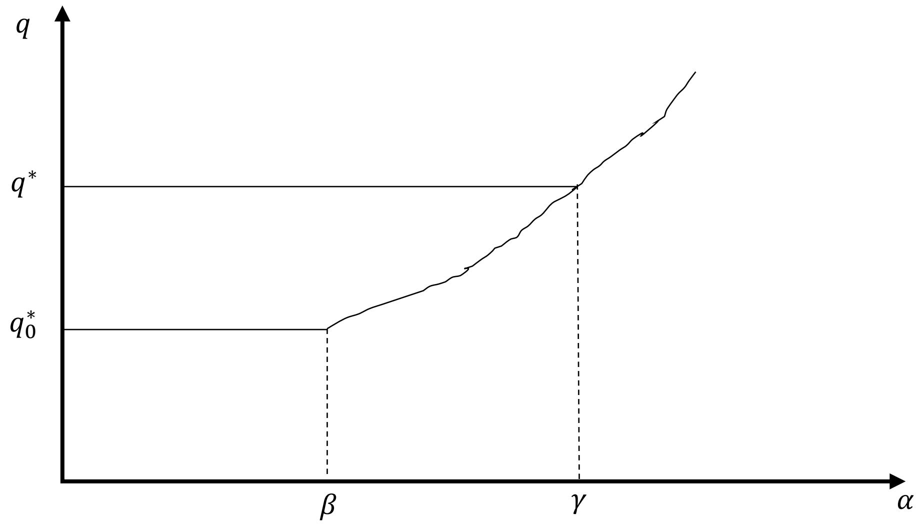

Fig. 2 Relationships between q^* and q_0^*

Now we explore the effect of bidirectional option contracts on the maximum expected profits of the retailer and the supplier, which leads to the following proposition.

Proposition 4 ***The maximum expected profits of the retailer and the supplier are both greater with bidirectional option contracts than without them.***

This proposition demonstrates that bidirectional option contracts benefit the retailer. The main reason is because the retailer has the right to obtain the additional products or return the partial leftovers by exercising the bidirectional options as the call or put options according to the realized market demand. Hence, the retailer can flexibly respond to the demand volatility, which results in an increase in his maximum expected profit. Furthermore, this proposition proves that bidirectional option contracts are also beneficial for the supplier. This is due to the reason that the supplier has the right to arrange the production schedule based on the retailer's order quantity of two types including the firm order and the bidirectional options order, and the stochastic market demand. Hence, the supplier can flexibly meet the irregular order, which leads to an increase in his maximum expected profit. In conclusion, the above proposition indicates that both the retailer and the supplier are better off in adopting bidirectional option contracts. Obviously, the usage of bidirectional option contracts can improve the supply chain members' individual performances and therefore achieve the win-win outcomes.

Recalling Zhao *et al.* (2013), they only consider the stochastic demand but neglect the service requirement. It indicates the relevant results in their paper, such as the retailer's optimal ordering policy without a service requirement, can be seemed as a special case for our study. In addition, they do not consider the variation on the retailer's maximum expected profit after adopting bidirectional option contracts. Thus, an important managerial implication for the retailer working in the circumstance that is similar to our study has not been pointed out. That is, bidirectional option contracts are more profitable than wholesale price contracts to the retailer. Furthermore, they never consider the supplier's optimal production policy, let alone the variation on the supplier's maximum expected profit after adopting bidirectional option contracts. Thus, our paper can be seems as an extension of their paper to a more complex and realistic supply chain environment.

5.3 The effect of the service level

Now we explore the effect of the service level on the maximum expected profits of the retailer and the supplier, which yields the following proposition.

Proposition 5 ***With bidirectional option contract, the retailer's maximum expected profit is a non-increasing function of the service requirement α while the supplier's maximum expected profit is a non-decreasing function of the service requirement α.***

This proposition shows that the service requirement made a substantial impact on the retailer's maximum expected profit. If the retailer has a desire to achieve higher expected profit, the customer has to face a lower service level. If the customer requires a higher service level, the retailer has to achieve lower expected profit. Obviously, it is important for the retailer to choose an appropriate service target. Furthermore, this proposition shows that the service requirement also has a significant impact on the supplier's maximum expected profit. If the supplier has a desire to achieve higher expected profit, the retailer faces a higher service level. If the retailer faces a lower service level, the supplier has to generate lower expected profit. Obviously, it is important for the supplier to induce the retailer to increase his order and aim for a higher service level.

We depict the relationships between $\pi_r(q_0^*)$, $\pi_r(q^*, q_1^*)$ and α in figure 3. From the following figure, we see that when the service requirement is not binding ($\alpha \leq \gamma$), the retailer's maximum expected profit with bidirectional option contracts is constant in α. When the service requirement is binding ($\alpha > \gamma$), the retailer's maximum expected profit with bidirectional option contracts is decreasing in α. Moreover, we see that with a service requirement the retailer's maximum expected profit with bidirectional option contracts is greater than without them. Hence, when there is a service requirement, the retailer's maximum expected profit without or with bidirectional option contracts is non-increasing in α.

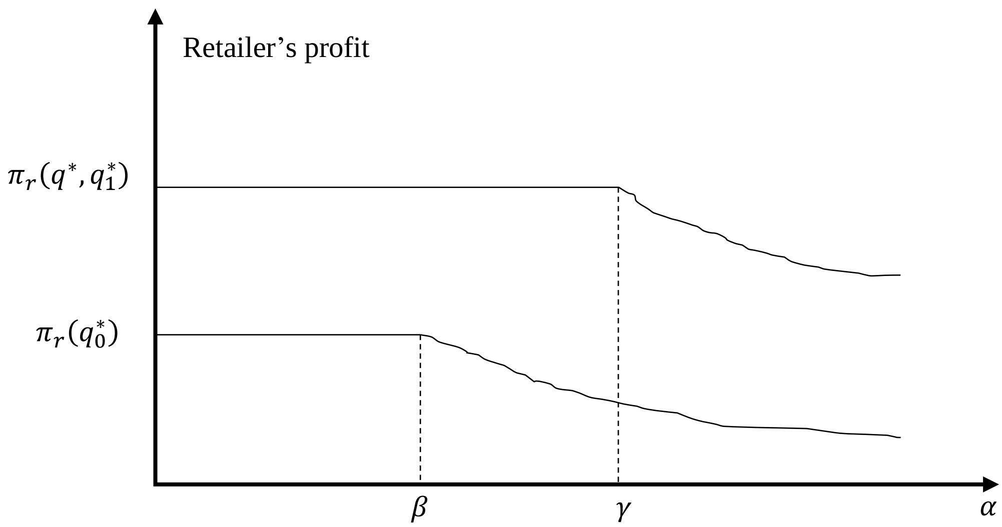

Fig. 3 Relationship between $\pi_r(q^*, q_1^*)$, $\pi_r(q_0^*)$ and α

We depict the relationships between $\pi_s(Q_0^*)$, $\pi_s(Q^*)$ and α in figure 4. From the following figure, we see that when the service requirement is not binding ($\alpha \leq \gamma$), the supplier's maximum expected profit with bidirectional option contracts is constant in α. When the service requirement is binding ($\alpha > \gamma$), the supplier's maximum expected profit with bidirectional option contracts is increasing in α. Moreover, we see that with a service requirement the supplier's maximum expected profit is greater with bidirectional option contracts than without them. Hence, when there is a service requirement, the supplier's maximum expected profit without or with bidirectional option contracts is non-decreasing in α.

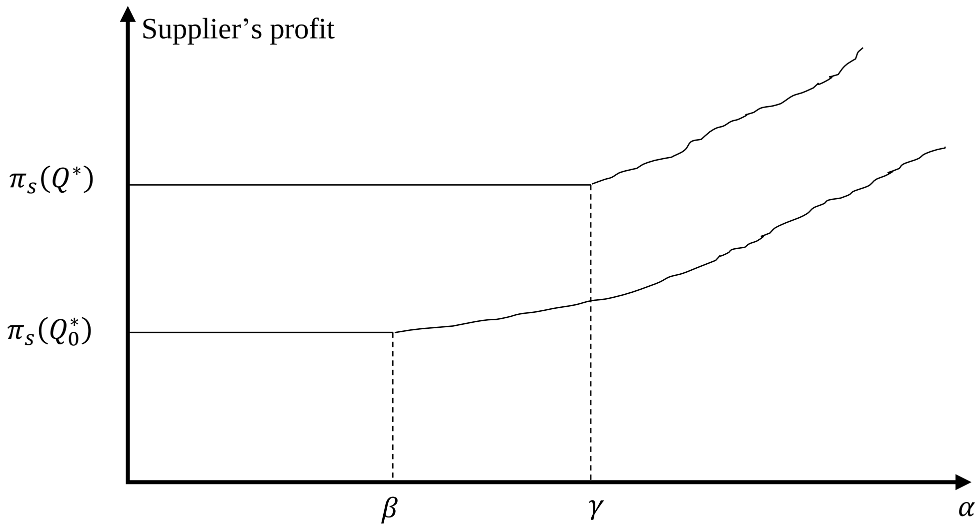

Fig. 4 Relationship between $\pi_s(Q^*)$, $\pi_s(Q_0^*)$ and α

Recalling Chen and Shen (2012), the similar optimization problems with a service requirement are taken into account. The major difference between their model and ours is that they consider call option contracts while we consider bidirectional option contracts. By comparing with the relevant results in the mentioned-above paper, the following conclusions can be derived. First, in comparison with wholesale price contracts, either call or bidirectional option contracts will increase the individual members' maximum expected profits. Next, with either call or bidirectional option contracts, the retailer's maximum expected profit is non-increasing in the service requirement α while the supplier's maximum expected profit is non-decreasing in the service requirement α. Such conclusions are sufficient to support the following point. That is, bidirectional option contracts are a natural extension of call option contracts.

6. Supply chain coordination

In this section, we address the problem of the supply chain coordination with bidirectional option contracts and a service requirement.

We take the members of the supply chain as an entity and study the centralized system's optimal production policy with a service requirement. We assume that the centralized system's production quantity is Q_I. Then, the expected profit of the centralized system, denoted $\Pi_I(Q_I)$, is

$$\Pi_I(Q_I) = pE[\min(Q_I, D)] + sE[(Q_I - D)^+] - cQ_I - gE[(D - Q_I)^+]$$

The first two terms are the expected revenue and the salvage value of unsold products. The last two terms capture the production costs and the penalty costs. Then,

$$\Pi_I(Q_I) = (p+g-c)Q_I - g\mu - (p+g-s)\int_0^{Q_I} F(x)dx \tag{17}$$

Thus, the decision problem faced by the centralized system in the presence of a service requirement is

$$\max_{Q_I \geq 0} \Pi_I(Q_I)$$

$$s.t.\ P_r\{Q_I \geq D\} \geq \alpha \tag{18}$$

From (18), we derive that $q \geq F^{-1}(\alpha)$ and $q^\alpha \equiv F^{-1}(\alpha)$. It's clear that q^α is an increasing function of α.

Lemma 3 ***Without a service requirement, the centralized system's expected profit $\boldsymbol{\Pi_I(Q_I)}$ is concave in $\boldsymbol{Q_I}$.***

From this lemma, we conclude that without a service requirement the centralized system has a unique optimal production policy. Then, without a service requirement the centralized system's optimal production quantity is

$$Q_I^\tau = F^{-1}\left(\frac{p+g-c}{p+g-s}\right) \tag{19}$$

Set $\tau = (p+g-c)/(p+g-s)$, which represents the maximum service level when there is no service requirement. Then, we obtain that $Q_I^\tau \equiv F^{-1}(\tau)$.

Regarding the optimal production quantity of the centralized system in the presence of a service requirement, the following proposition is obtained.

Proposition 6 ***With a service requirement, the centralized system's optimal production quantity is***

$$\boldsymbol{Q_I^*} = \begin{cases} \boldsymbol{Q_I^\tau} & \boldsymbol{if\ \alpha \leq \tau} \\ \boldsymbol{q^\alpha} & \boldsymbol{if\ \alpha > \tau} \end{cases} \tag{20}$$

From this proposition, we find that with a service requirement the centralized system's optimal production quantity is an interval. When the service requirement are not binding ($\alpha \leq \gamma$), we obtain that $Q_I^* = Q_I^\tau$. In this case, we derive that $\Pi_I(Q_I^*)$ is a constant function of α. When the service requirement are binding ($\alpha > \gamma$), we obtain that $Q_I^* = q^\alpha$. In this case, we derive that $\Pi_I(Q_I^*)$ is a decreasing function of α. In conclusion, with a service requirement the centralized system's maximum expected profit is a non-increasing function of α. We have the following insight.

Proposition 7 ***With a service requirement, The centralized system's optimal service level is $\boldsymbol{\alpha^* = \frac{p+g-c}{p+g-s}}$.***

The above proposition shows that the optimal service level is the maximum service level corresponding to the centralized system's optimal production quantity without a service requirement.

Subsequently, we consider the coordination condition for a supply chain with bidirectional option contracts and a service requirement. The following proposition is derived.

Proposition 8 ***With a service requirement, Portfolio contracts with bidirectional options that satisfy*** $\boldsymbol{w_1 = w_2 - b + \frac{2(1-\alpha)(p+g-s)(p-w_2)}{g-s}}$ ***can coordinate the supply chain.***

This proposition shows that the coordination condition of a supply chain with bidirectional option contracts and a service requirement is determined by the unit retail price, the unit wholesale price, the unit purchase and exercise prices of bidirectional options. Moreover, we see that this coordination condition is independent with the probability density function for the stochastic market demand, that is a *distribution-free* coordination condition. Thus, this feature makes this coordination condition easier to be implemented in practice. Furthermore, we see that the expected profits of both the retailer and the supplier with the coordinating contracts are at least the same as those without. Obviously, when there is a service requirement, there always exists a Pareto improvement with bidirectional option contracts.

7. Conclusions

In this study, we explore the procurement and production problems for a perishable supply chain with bidirectional option contracts and a service requirement. We take into consideration a supplier-retailer supply chain system, in which the supplier manufactures a type of short life products and the retailer, who commits to a service target to the customers and satisfies the unanticipated demand through the firm order and the bidirectional options order. To the best of authors' knowledge, this is the first research that examines the value of bidirectional option contracts and service level to the supply chain risk management. Our work provides several interesting observations.

Observation 1: With bidirectional option contracts, there is an optimal ordering policy for the retailer in the presence of a service requirement. In a similar condition, there also exists an optimal production policy for the supplier. Therefore, the members of a supply chain can adopt the appropriate decision policies based on our findings to maximize their expected profits with bidirectional option contracts and a required service level.

Observation 2: Our finding shows that the adoption of bidirectional option contracts has a significant impact on the optimal decision policies and the maximum expected profits of both the retailer and the supplier in the presence of a required service level. By benchmarking the case without bidirectional option contracts, we prove that the total order quantity will be increased by the retailer, the production will be arranged more flexibly, and more benefit will be gained by the two parties after using bidirectional option contracts in the presence of a service requirement. Hence, when there is a service requirement, the members of a supply chain can better optimize their decisions and enhance their profits through the adoption of bidirectional option contracts.

Observation 3: Our finding also shows that the service level has a significant impact on the maximum expected profits of both the retailer and the supplier with or without bidirectional option contracts. With or without bidirectional option contracts, the maximum expected profit of the retailer is non-increasing in the service requirement while that of the supplier is non-decreasing in the service requirement. Hence, with or without bidirectional option contracts, the members of a supply chain must set an appropriate service target to balance the conflicts of interest in the presence of service requirement.

Observation 4: We find that when there is a service requirement, the adoption of bidirectional option contracts can coordinate the channel. At this point, the coordination condition is independent with market demand distribution. Therefore, without the knowledge of the market demand distribution, the members can achieve the highest profits and the chain can achieve the maximum efficiency. This feature makes the implementation of the coordinating contracts easier in practice.

This research makes several key contributions as follows. (1) Our work extends the existing literature on bidirectional option contracts by incorporating the service requirement into the model. We explore the effect of the service level on the supply chain with bidirectional option contracts. (2) Our research is different from most existing literature referring to bidirectional option contracts that only focus on its effect on the retailer's ordering behavior. We assume that the supplier can arrange his production considering bidirectional option contracts by setting a parameter g which denotes the supplier's unit penalty cost involved in accelerating production or obtaining products from an alternative source. We explore the supplier's production behavior with bidirectional option contracts and a required service level. We also examine the value of bidirectional option contracts on the performance of both the retailer and the supplier in the presence of a required service level. (3) The supplier is always assumed to use the make-to-order production strategy in most existing literature. At this point, the supply chain is coordinated through the unilateral coordination mechanism from the perspective of the retailer. However, with bidirectional option contracts the supplier can arrange his production schedule based on profit maximization principle. Our study complements to the existing literature by designing the bilateral coordination mechanism from the perspective of both the retailer and the supplier.

There are some future research directions that are worth considering. (1) it will be interesting to explore the effect of different types of option contracts on the supply chain and consider which type of option contracts is the best choice when there is a service requirement. (2) Both the retailer and the supplier are assumed to be risk-neutral in this paper. Another interesting direction is to consider different risk attitudes of the members, such as loss-averse supplier (Luo and Chen, 2015) or loss-averse retailer (Chen and Zhou, 2015). (3) In this paper, we assume that the dominant position is taken by the supplier. Considering different supply chain power structures (Chen *et al.* 2015; Chen and Wang, 2015) is another interesting research avenue to be explored. (4) Similar to Zhao *et al.* (2013), in our study we consider the unit exercise price under the call options equals to that under the put options. Obviously, a particularly interesting issue is to consider separate parameters for exercising bidirectional options as call or put options.

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Appendix

Proof of Lemma 1

From (1), we derive that $\frac{\partial \pi_r(q_1,q_2)}{\partial q_1} = (p-w_1)-(p-w_2)F(q_1+q_2)-(w_2-s)F(q_1-q_2)$, $\frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_1^2} = -(p-w_2)f(q_1+q_2)-(w_2-s)f(q_1-q_2)<0$, $\frac{\partial \pi_r(q_1,q_2)}{\partial q_2} = (p-w_2-b)-(p-w_2)F(q_1+q_2)+(w_2-s)F(q_1-q_2)$, $\frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_2^2} = -(p-w_2)f(q_1+q_2)-(w_2-s)f(q_1-q_2)<0$. Since $\frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_2 \partial q_1} = -(p-w_2)f(q_1+q_2)+(w_2-s)f(q_1-q_2)$, we derive that $\begin{vmatrix} \frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_1^2} & \frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_r(q_1,q_2)}{\partial q_1^2} \end{vmatrix} = 4(p-w_2)(w_2-s)f(q_1+q_2)f(q_1-q_2)>0$. Obviously, the Hessian matrix of $\pi_r(q_1,q_2)$ is negative definite. Hence, $\pi_r(q_1,q_2)$ is jointly concave in q_1 and q_2.

Proof of Corollary 1

Recalling (2), the retailer's maximum expected profit with bidirectional option contracts in the presence of service requirement is

$$\pi_r(q_1^\gamma, q_2^*) = (p-w_1)q_1^\gamma + (p-w_2-b)q_2^* - (p-w_2)\int_0^{q_1^\gamma+q_2^*} F(x)dx - (w_2-s)\int_0^{q_1^\gamma-q_2^*} F(x)\,dx$$

Then we derive $\frac{d\pi_r(q_1^\gamma,q_2^*)}{dw_1} = -q_1^\gamma + (p-w_1)\frac{dq_1^\gamma}{dw_1} + (p-w_2-b)\frac{dq_2^*}{dw_1} - (p-w_2)F(q_1^\gamma+q_2^*)\frac{d(q_1^\gamma+q_2^*)}{dw_1} - (w_2-s)F(q_1^\gamma-q_2^*)\frac{d(q_1^\gamma-q_2^*)}{dw_1}$.

(1)When $\alpha \le \gamma$, we obtain that $q^* = q^\gamma$ and $q_2^* = q^\gamma - q_1^\gamma$. Then

$$\frac{d\pi_r\left(q_1^\gamma,q_2^*\right)}{dw_1}=-q_1^\gamma+\left[(w_2+b-w_1)-2(w_2-s)F\left(2q_1^\gamma-q^\gamma\right)\right]\frac{dq_1^\gamma}{dw_1}+\left[(p-w_2-b)-(p-w_2)F(q^\gamma)+(w_2-s)F\left(2q_1^\gamma-q^\gamma\right)\right]\frac{dq^\gamma}{dw_1}=-q_1^\gamma<0$$

In this case, $\pi_r\left(q_1^\gamma,q_2^*\right)$ is a decreasing function of w_1.

(2)When $\alpha>\gamma$, we obtain that $q^*=q^\alpha$ and $q_2^*=q^\alpha-q_1^\gamma$. Then

$$\frac{d\pi_r\left(q_1^\gamma,q_2^*\right)}{dw_1}=-q_1^\gamma+\left[(w_2+b-w_1)-2(w_2-s)F\left(2q_1^\gamma-q^\alpha\right)\right]\frac{dq_1^\gamma}{dw_1}$$

$$<-q_1^\gamma+\left[(w_2+b-w_1)-2(w_2-s)F\left(2q_1^\gamma-q^\gamma\right)\right]\frac{dq_1^\gamma}{dw_1}=-q_1^\gamma<0$$

In this case, $\pi_r\left(q_1^\gamma,q_2^*\right)$ is also a decreasing function of w_1.

Proof of Lemma 2

From (8), we derive that $\frac{d\pi_S(Q)}{dQ}=(g-c)-(g-s)F(Q)$ and $\frac{d^2\pi_S(Q)}{dQ^2}=-(g-s)f(Q)<0$. Obviously, $\pi_S(Q)$ is concave in Q.

Proof of Corollary 2

Recalling (9), the supplier's maximum expected profit with bidirectional option contracts in the presence of service requirement is

$$\pi_S(Q^*)=(w_2+b-g)q^*+(w_1-w_2-b)q_1^\gamma-(w_2-g)\int_0^{q^*}F(x)\,dx+(w_2-s)\int_0^{2q_1^\gamma-q^*}F(x)\,dx$$
$$+(g-c)Q^*-(g-s)\int_0^{Q^*}F(x)dx$$

When $\alpha>\gamma$, we obtain that $q^*=q^\alpha$. Then

$$\frac{d\pi_S(Q^*)}{dw_1}=q_1^\gamma-\left[(w_2+b-w_1)-2(w_2-s)F\left(2q_1^\gamma-q^\alpha\right)\right]\frac{dq_1^\gamma}{dw_1}+(g-s)\left[F\left(Q^\lambda\right)-F(Q^*)\right]\frac{dQ^*}{dw_1}$$

$$>q_1^\gamma-\left[(w_2+b-w_1)-2(w_2-s)F\left(2q_1^\gamma-q^\gamma\right)\right]\frac{dq_1^\gamma}{dw_1}+(g-s)\left[F\left(Q^\lambda\right)-F(Q^*)\right]\frac{dQ^*}{dw_1}$$

$$=q_1^\gamma+(g-s)\left[F\left(Q^\lambda\right)-F(Q^*)\right]\frac{dQ^*}{dw_1}$$

If $Q^\lambda\le q_1^\gamma$, we obtain that $Q^*=q_1^\gamma$, $\frac{dQ^*}{dw_1}<0$ and $F\left(Q^\lambda\right)\le F(Q^*)$. In this case, we derive that $\frac{d\pi_S(Q^*)}{dw_1}>0$. If $q_1^\gamma<Q^\lambda<q^*$, we obtain that $Q^*=Q^\lambda$, $\frac{dQ^*}{dw_1}=0$ and $F\left(Q^\lambda\right)=F(Q^*)$. In this case, we derive that $\frac{d\pi_S(Q^*)}{dw_1}>0$. If $Q^\lambda\ge q^*$, we obtain that $Q^*=q^*$, $\frac{dQ^*}{dw_1}=0$ and $F\left(Q^\lambda\right)>F(Q^*)$. In this case, we derive that $\frac{d\pi_S(Q^*)}{dw_1}>0$.

Proof of Proposition 3

Since $b < \frac{(p-w_1)(w_2-s)+(p-w_2)(w_1-s)}{p-s}$, we can derive that $\gamma > \beta$. If $\alpha \le \beta$, we obtain that $q^* = q^\gamma$ and $q_0^* = q_0^\beta$. Since $q^\gamma > q_0^\beta$, it follows that $q^* > q_0^*$. If $\beta < \alpha < \gamma$, we obtain that $q^* = q^\gamma$ and $q_0^* = q^\alpha$. Since q^α is an increasing function of α, $q^\alpha|_{\alpha=\beta} = q_0^\beta$ and $q^\alpha|_{\alpha=\gamma} = q^\gamma$, it follows that $q^* > q_0^*$. If $\alpha \ge \gamma$, we obtain that $q^* = q^\alpha$ and $q_0^* = q^\alpha$. It follows that $q^* = q_0^*$.

Proof of Proposition 4

First, we study the effect of bidirectional option contracts on the retailer's maximum expected profit.

Set $\Delta(q_2^*) = \pi_r(q_0^*, q_2^*) - \pi_r(q_0^*)$. Using (5) and (14), we derive that $\Delta(q_2^*) = (p - w_2 - b)q_2^* - (p - w_2)\int_0^{q_0^*+q_2^*} F(x)dx - (w_2 - s)\int_0^{q_0^*-q_2^*} F(x)\,dx + (p - s)\int_0^{q_0^*} F(x)dx$.

(1)When $\alpha \le \beta$, we obtain $q_0^* = q_0^\beta$ and $q^* = q^\gamma$. Then, $\Delta(q_2^*) = (p - w_2 - b)q_2^* - (p - w_2)\int_0^{q_0^\beta+q_2^*} F(x)dx - (w_2 - s)\int_0^{q_0^\beta-q_2^*} F(x)dx + (p - s)\int_0^{q_0^\beta} F(x)dx$

Since $\Delta(0) = 0$ and $\frac{d\Delta(q_2^*)}{dq_2^*}|_{q_2^*=0} = (p - w_2)[F(q^\gamma) - F(q_0^\beta)] - (w_2 - s)[F(2q_1^\gamma - q^\gamma) - F(q_0^\beta)] > 0$, we can deduce that $\Delta(q_2^*) > 0$. It follows that $\pi_r(q_0^\beta, q_2^*) > \pi_r(q_0^\beta)$. Hence $\pi_r(q_1^\gamma, q_2^\gamma) > \pi_r(q_0^\beta)$.

(2)When $\beta < \alpha \le \gamma$, we obtain $q_0^* = q^\alpha$ and $q^* = q^\gamma$. Then, $\Delta(q_2^*) = (p - w_2 - b)q_2^* - (p - w_2)\int_0^{q^\alpha+q_2^*} F(x)dx - (w_2 - s)\int_0^{q^\alpha-q_2^*} F(x)dx + (p - s)\int_0^{q^\alpha} F(x)dx$. Since $\Delta(0) = 0$ and $\frac{d\Delta(q_2^*)}{dq_2^*}|_{q_2^*=0} = (p - w_2)[F(q^\gamma) - F(q^\alpha)] - (w_2 - s)[F(2q_1^\gamma - q^\gamma) - F(q^\alpha)] > 0$, we can deduce that $\Delta(q_2^*) > 0$. It follows that $\pi_r(q^\alpha, q_2^*) > \pi_r(q^\alpha)$. Hence $\pi_r(q_1^\gamma, q_2^\gamma) > \pi_r(q^\alpha)$.

(3)When $\alpha > \gamma$, we obtain $q_0^* = q^\alpha$ and $q^* = q^\alpha$. Similarly to (2), we derive that $\pi_r(q_1^\gamma, q_2^\gamma) > \pi_r(q^\alpha)$.

Now we study the effect of bidirectional option contracts on the supplier's maximum expected profit.

When $\alpha > \gamma$, we obtain that $q^* = q^\alpha$ and $q_0^* = q^\alpha$. Set $\Delta(w_1) = \pi_s(Q^*) - \pi_s(Q_0^*)$. Using (10) and (16), we derive that $\Delta(w_1) = (w_2 + b - g)q^\alpha + (w_1 - w_2 - b)q_1^\gamma - (w_2 - g)\int_0^{q^\alpha} F(x)\,dx + (w_2 - s)\int_0^{2q_1^\gamma-q^\alpha} F(x)\,dx + (g - c)Q^* - (g - s)\int_0^{Q^*} F(x)dx - (w_1 - c)q^\alpha$. Set $q_1^\gamma|_{w_1=w_1^0} = q^\alpha$. If $w_1 = w_1^0$ then $q_2^\gamma = 0$, $q^\gamma = q_1^\gamma = q^\alpha$ and $Q^* = q^\alpha$, which yields $\Delta(w_1^0) = 0$. Notice $\frac{d\Delta(w_1)}{dw_1}|_{w_1=w_1^0} > (g - s)[F(Q^\lambda) - F(Q^*)]\frac{dQ^*}{dw_1}$. If $Q^\lambda \le q_1^\gamma$, we obtain that $F(Q^\lambda) \le F(Q^*)$ and $\frac{dQ^*}{dw_1} < 0$, which yields

$\frac{d\Delta(w_1)}{dw_1}|_{w_1=w_1^0} > 0$. Hence $\pi_s(Q^*) > \pi_s(Q_0^*)$. If $Q^\lambda > q_1^\gamma$, we obtain that $\frac{dQ^*}{dw_1} = 0$, which yields $\frac{d\Delta(w_1)}{dw_1}|_{w_1=w_1^0} > 0$. Hence $\pi_s(Q^*) > \pi_s(Q_0^*)$. Hence, $\pi_s(Q^*) > \pi_s(Q_0^*)$. Similarly, when $\beta < \alpha \le \gamma$ and $\alpha \le \beta$ it follows that $\pi_s(Q^*) > \pi_s(Q_0^*)$.

Proof of Proposition 5

First, we consider the effect of service level on the retailer's maximum expected profit.

When $\alpha \le \gamma$, we obtain that $q^* = q^\gamma$ and $q_2^* = q^\gamma - q_1^\gamma$. In this case, we derive that $\pi_r(q_1^\gamma, q_2^*)$ is a constant function of α. When $\alpha > \gamma$, we obtain that $q^* = q^\alpha$ and $q_2^* = q^\alpha - q_1^\gamma$. Then

$$\frac{d\pi_r(q_1^\gamma, q_2^*)}{d\alpha} = \left[(p - w_2 - b) - (p - w_2)F(q^\alpha) + (w_2 - s)F\left(2q_1^\gamma - q^\alpha\right)\right]\frac{dq^\alpha}{d\alpha}$$

$$= \left\{(p - w_2)[F(q^\gamma) - F(q^\alpha)] - (w_2 - s)\left[F\left(2q_1^\gamma - q^\gamma\right) - F\left(2q_1^\gamma - q^\alpha\right)\right]\right\}\frac{dq^\alpha}{d\alpha}$$

Since $F(q^\gamma) < F(q^\alpha)$, $F\left(2q_1^\gamma - q^\gamma\right) > F\left(2q_1^\gamma - q^\alpha\right)$ and $\frac{dq^\alpha}{d\alpha} > 0$, we obtain that $\frac{d\pi_r(q_1^\gamma, q_2^*)}{d\alpha} < 0$. In this case, we derive that $\pi_r(q_1^\gamma, q_2^*)$ is a decreasing function of α.

Now we consider the effect of service level on the supplier's maximum expected profit.

When $Q^\lambda < q^*$, we derive that $\pi_s(Q^*)$ is a constant function of α. When $Q^\lambda \ge q^*$, we obtain $Q^* = q^*$. If $\alpha \le \gamma$ and $Q^\lambda \ge q^\gamma$, we obtain $Q^* = q^\gamma$. In this case, we derive that $\pi_s(Q^*)$ is a constant function of α. If $\alpha > \gamma$ and $Q^\lambda \ge q^\alpha$, we obtain $Q^* = q^\alpha$. Then $\frac{d\pi_s(Q^*)}{d\alpha} = (g - s)\left[F\left(Q^\lambda\right) - F(q^\alpha)\right]\frac{dq^\alpha}{d\alpha}$. Since $F\left(Q^\lambda\right) \ge F(q^\alpha)$ and $\frac{dq^\alpha}{d\alpha} > 0$, we obtain $\frac{d\pi_s(q^\alpha)}{d\alpha} \ge 0$. In this case, we derive that $\pi_s(Q^*)$ is a increasing function of α.

Proof of Lemma 3

From (17), we derive that $\frac{d\Pi_I(Q_I)}{dQ_I} = (p + g - c) - (p + g - s)F(Q_I)$ and $\frac{d^2\Pi_I(Q_I)}{d{Q_I}^2} = -(p + g - s)f(Q_I) < 0$. Obviously, $\Pi_I(Q_I)$ is strictly concave in Q_I.

Proof of Proposition 8

Both the retailer's order quantity and the supplier's production quantity need coordinate. Only when $\lambda = \gamma$ is satisfied, the supplier has a motivation to manufacture the same quantity as in the coordinated supply chain. Then, we obtain that $\frac{g-c}{g-s} = \frac{2p-w_1-w_2-b}{2p-2w_2}$. Since $\tau = \frac{p+g-c}{p+g-s} > \frac{g-c}{g-s} = \lambda$, we obtain that $\tau > \lambda \equiv \gamma$. Therefore, only when $\alpha = \frac{p+g-c}{p+g-s}$ is satisfied, the retailer has a motivation to purchase the same quantity as in the coordinated supply chain. With some algorithm, we get $w_1 = w_2 - b + \frac{2(1-\alpha)(p+g-s)(p-w_2)}{g-s}$, which is the sufficient condition for the channel coordination with bidirectional option contracts and a service requirement.